Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS

2C/2D

Calculator-assumed

WACE Examination 2012

Marking Key

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

When examiners design an examination, they develop provisional marking keys that can be reviewed at a marking key ratification meeting and modified as necessary in the light of candidate responses.

Question 7 **(4 marks)**

Given $C = 6.2 \times 10^{-2}$ and $D = 7.7 \times 10^{-3}$, determine the value of:

(a) $C \times D$. Give the answer in scientific notation. (2 marks)

Solution
$6.2 \times 10^{-2} \times 7.7 \times 10^{-3}$ $= 4.774 \times 10^{-4}$
Specific behaviours
✓ calculates multiplication correctly ✓ expresses answer in scientific notation correctly

(b) $C - D$. Give the answer to **two (2)** significant figures. (2 marks)

Solution
$C - D = 0.0543$ $= 0.054$ to two significant figures.
Specific behaviours
✓ calculates subtraction correctly ✓ expresses answer correct to two significant figures

Question 8 **(6 marks)**

From 1 July 2011, these fees (including GST) are recommended by the Real Estate Institute of Tasmania.

Real Estate Fees/Commission in Tasmania

Selling price of each property	Real estate fees
Does not exceed $10 000	10.73% with a minimum of $100
From $10 001 – $50 000	$1073 plus 5.85% of excess over $10 000
From $50 001 – $100 000	$3413 plus 4.10% of excess over $50 000
From $100 001 upwards	$5463 plus 3.88% of excess over $100 000

Jenny owns several properties in Tasmania.

(a) If Jenny sells a house for $320 000, how much does she have to pay in real estate fees? (3 marks)

Solution
$\$\left[5463+\frac{3.88}{100}\times(320\ 000-100\ 000)\right]=\$[5463+8536]$ $=\$13\ 999$
Specific behaviours
✓ identifies correct price bracket ✓ determines correct fee for each part ✓ calculates the total fee paid

(b) Jenny wants to reduce the number of properties she owns. She is trying to decide whether to sell a house for $320 000 or sell a group of four apartments, for $80 000 each. Which option will result in her paying the smaller amount of real estate fees and how much less will she pay in fees with this option? (3 marks)

Solution
One apartment: $\text{Fee}=\$\left[3413+\frac{4.10}{100}(80\ 000-50\ 000)\right]$ $=\$[3413+1230]$ $=\$4643$ Total fees for four units is $18 572 It would be cheaper to sell a single house. Saving $4573 in fees.
Specific behaviours
✓ calculates the fee for a single apartment ✓ calculates the fee for four apartments ✓ chooses correct option and savings

Question 9 **(6 marks)**

Marine researchers were interested in estimating the population of sea lions in one of the breeding colonies on the west coast of Western Australia. One day, they observed and tagged 23 sea lions in the colony. After allowing time for these tagged sea lions to mix with the others, they observed a sample of 20 sea lions and found that seven of them had tags.

(a) Use the capture-recapture technique to estimate the total population of the sea lion colony. (2 marks)

Solution
$\frac{23}{x} = \frac{7}{20}$ $x = 65.71 \approx 66$ (also accept 65 and 70)
Specific behaviours
✓ estimates population correctly (i.e. a whole number value) ✓ demonstrates use of capture-recapture technique

Three days later, the marine researchers observed a sample of 24 sea lions, and found that five of these were tagged.

(b) Use this additional information to form a better estimate of the total population of the sea lion colony. (2 marks)

Solution
$\frac{23}{x} = \frac{5}{24}$ $x = 110.4 \approx 110$ Better estimate: $\frac{66+110}{2} = 88$
Specific behaviours
✓ estimates population based on the second sample ✓ estimates population based on average of both samples

The researchers involved in this study were challenged by other marine researchers, who claimed that the population estimate was inaccurate because some of the sea lions originally tagged had lost their tags.

(c) (i) If this was the case, would such a claim imply a higher or lower estimate of the sea lion population than that calculated in Part (b)? (1 mark)

Solution
Lower estimate
Specific behaviours
✓ chooses correct estimate

(ii) Justify your answer to Part (c) (i). (1 mark)

Solution
In the recapture, there should have been more sea lions with tags, and hence a greater proportion of tagged sea lions in the recapture and therefore lower overall population.
Specific behaviours
✓ justifies choice based on valid reasoning (accept mathematical reasoning)

Question 10 **(7 marks)**

The table below produced by the Australian Bureau of Statistics shows results compiled from Household Use of Information Technology (HUIT) data collected by the Multipurpose Household Survey (MPHS) for 2010–11.

Household Use of Information Technology, Australia, 2010–11

Households (in thousands)	Home internet access		Households with home internet access by frequency of access			
	Households without home internet access	Households with home internet access	Every day	At least weekly	At least monthly	Never/ Don't know
New South Wales	568	2164	1666	411	43	44
Victoria	458	1684	1280	354	34	16
Queensland	355	1347	1037	269	40	1
South Australia	160	497	373	109	10	5
Western Australia	163	714	557	129	21	7
Tasmania	62	146	110	29	4	3
Northern Territory	14	51	39	9	2	1
Australian Capital Territory	16	122	98	21	1	2
Total households	1796	6725	5160	1331	155	79

Note: All data are rounded.

(a) Determine the total number of households surveyed. (2 marks)

Solution
8 521 000
Specific behaviours
✓ identifies the correct parts from the table to determine the total ✓ expresses the correct total in thousands

A household is randomly selected from those surveyed.

(b) Determine the probability that the household had home internet access. (2 marks)

Solution
$\frac{6\ 725\ 000}{8\ 521\ 000} = 0.7892$
Specific behaviours
✓ identifies correct numerator ✓ identifies correct denominator

(c) Determine the probability that the household is in Western Australia and accessed the internet at home every day. (1 mark)

Solution
$\frac{557}{8521} = 0.0654$
Specific behaviours
✓ determines the correct probability

(d) Given that the household had internet access, determine the probability that it was in Victoria. (2 marks)

Solution
$\frac{1684}{6725} = 0.25$
Specific behaviours
✓ identifies correct numerator ✓ identifies correct denominator

Question 11 **(8 marks)**

A city is to host an economic forum to be attended by the leaders of a number of nations. To ensure the safety of the leaders a sophisticated communication network, linking a number of control points, is to be set up. There are eight control points P_1, P_2, ..., P_8 and the costs, in thousands of dollars, of establishing a direct link between points is given in the following table.

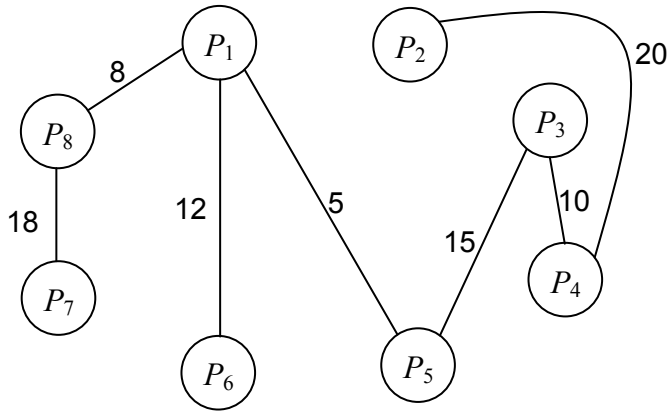
	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
P_1	-	25	18	16	5	12	19	8
P_2	25	-	22	20	24	25	22	23
P_3	18	22	-	10	15	16	20	21
P_4	16	20	10	-	19	21	28	27
P_5	5	24	15	19	-	15	20	10
P_6	12	25	16	21	15	-	28	30
P_7	19	22	20	28	20	28	-	18
P_8	8	23	21	27	10	30	18	-

The city needs to establish a minimal cost network.

(a) Use Prim's algorithm determine the minimal cost. (4 marks)

Solution
Starting at P_1 select the links: P_1P_5, P_1P_8, P_1P_6, P_3P_5, P_3P_4, P_8P_7, P_4P_2 minimal cost: \$88 000
Specific behaviours
✓ determines three (3) or more correct links ✓ determines five (5) or more correct links ✓ determines seven (7) correct links ✓ calculates correct minimal cost

(b) Represent the solution found in Part (a) as a network. (2 marks)



Solution
See diagram
Specific behaviours
✓ identifies at least four links and costs (based on Part (a))
✓ identifies all links and costs (based on Part (a))

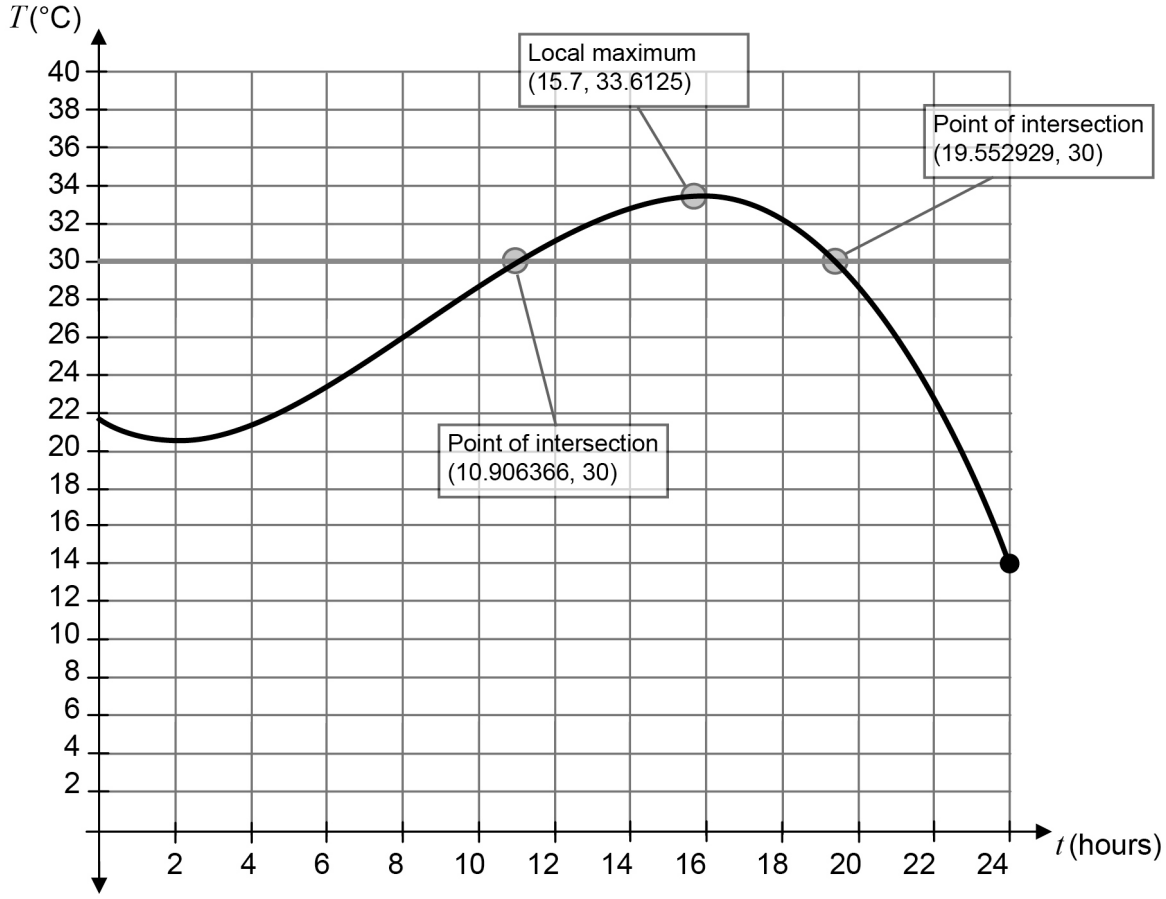
(c) The planners realise that the cost of connecting the control points P_1 and P_2 can be reduced by \$8000 by using a remote device. By how much does the use of the remote device reduce the minimum cost of constructing the network? Justify your solution.
(2 marks)

Solution
Cost of link P_1P_2 becomes \$17 000. Add link P_1P_2 and delete link P_2P_4 the cost will be reduced by \$3000.
Specific behaviours
✓ determines the reduction of \$3000
✓ justifies solution correctly

Question 12 **(9 marks)**

Hanna used her calculator to determine a function that modelled the temperature T (in °C) on one day last summer. Her temperature function was $T = -0.01t^3 + 0.266t^2 - 0.957t + 21.77$. The function applied from midnight ($t = 0$) to midnight ($t = 24$).

(a) On the axes below sketch the temperature function for the 24 hour period. (4 marks)



Solution
See graph
Specific behaviours
✓ identifies correct shape of cubic function ✓ evaluates correct end point i.e. $t = 0$ and $t = 24$ values ✓ shows local minimum ✓ shows local maximum

(b) What did Hanna's function suggest the temperature was at 5.00 pm? (1 mark)

Solution
33.2° (or a reasonable value according to sketch in Part (a)).
Specific behaviours
✓ determines the correct value from graph in Part (a).

(c) The actual maximum temperature of 35.1 °C occurred at 1.00 pm. Comment on how accurate her function was for modelling the maximum temperature. (2 marks)

Solution
According to the sketch the maximum temperature of 33.6 °C occurred at 15.7 h (or 3.42 pm). At 1.00 pm the sketch/function indicates a temperature of 32.3 °C. Model is not too accurate.
Specific behaviours
✓ recognises appropriate level of accuracy ✓ provides justification for choice

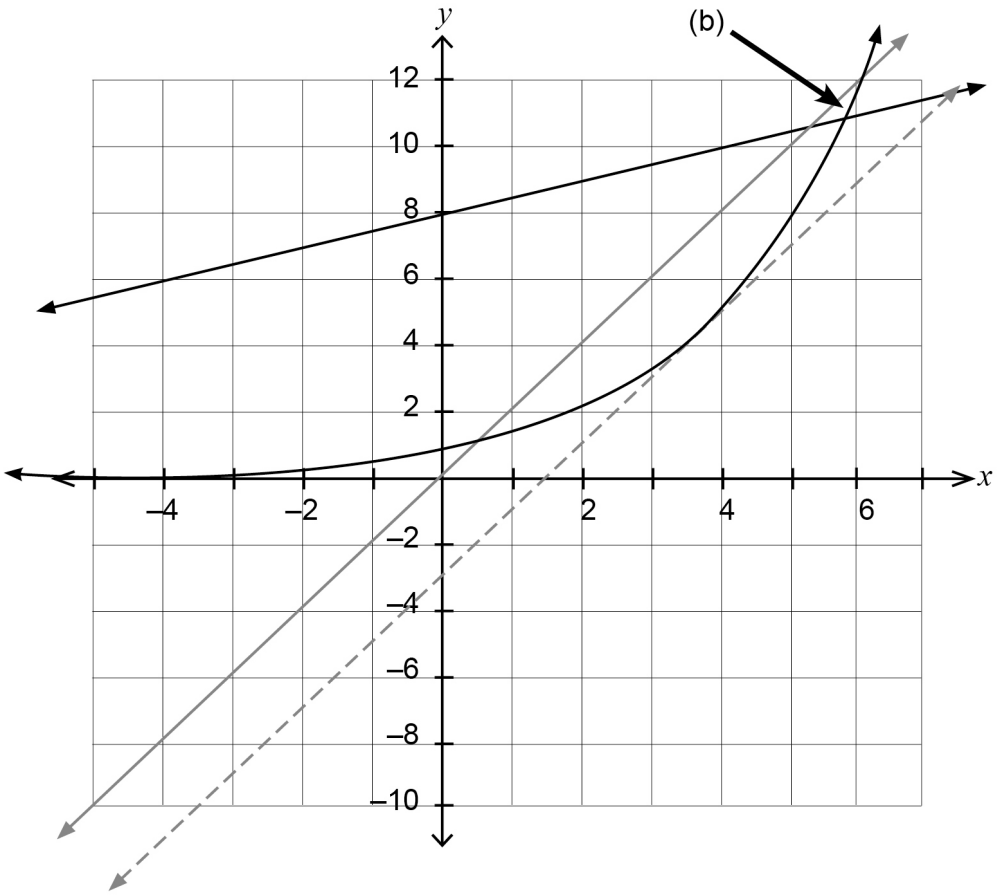
(d) According to the function for what percentage of the 24 hour period was the temperature above 30 °C?

(2 marks)

Solution
$\frac{19.55-10.9}{24}$ $\frac{8.65}{24}\times 100 = 36.04\%$
Specific behaviours
✓ identifies the correct proportion $\frac{8.65}{24}$ (Accept $\frac{8.6}{24}$) ✓ expresses solution as a percentage

Question 13 **(11 marks)**

The graph of $y = \frac{x}{2} + 8$ has been drawn on the axes below.



(a) Sketch the graph of $y = 1.5^x$ on the axes given. (4 marks)

Solution
See graph
Specific behaviours
✓ identifies correct shape ✓ identifies correct behaviours for extreme values of x ✓ identifies correct behaviours as $x \to \pm\infty$ (uses arrows) ✓ identifies correct y-intercept

(b) Use the graph to estimate the solution of the equation $1.5^x = \frac{x}{2} + 8.$ Show on your graph where you found the solution. (2 marks)

Solution
From graph, $x = 5.9$ (value consistent with sketch)
Specific behaviours
✓ identifies the correct solution ✓ shows the solution on the graph

(c) Use your calculator to determine the second solution to the equation $1.5^x = \frac{x}{2} + 8.$.

(2 marks)

Solution
$-15.99695136 \approx -15.997$ (accept -16)
Specific behaviours
✓ calculates correct magnitude of solution ✓ communicates answer with appropriate degree of accuracy

(d) The graphs of $y = 1.5^x$ and $y = 2x + c$ are to have one point of intersection.

Which would be the better estimate of c such that the graphs of $y = 1.5^x$ and $y = 2x + c$ have only one point of intersection: $c = 3$, $c = 0$, $c = -3$? Explain your answer. (3 marks)

Solution
$c = -3$ When $c = 3$, and $c = 0$, there are two points of intersection, but when $c = -3$ there is only one point of intersection
Specific behaviours
✓ choose correct value of c ✓ makes reference to the number of points of intersection for where $c = 3$, and $c = 0$ ✓ compares number of points of intersection for $c = 3$, $c = 0$, $c = -3$

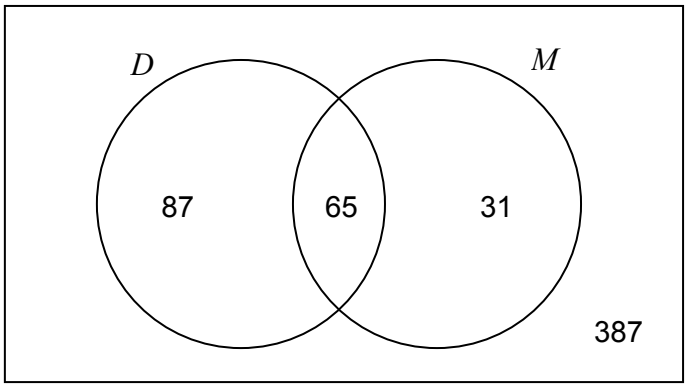
Question 14 **(11 marks)**

When travelling from Perth to London, passengers need to take two flights. Unfortunately, the first flight is sometimes delayed, and this can result in passengers missing their second, connecting flight.

Of 570 passengers who travelled from Perth to London on two flights, 152 had their first flight delayed. Of these delayed passengers, 65 missed their second, connecting flight. A total of 96 passengers missed their second, connecting flight.

Let D represent the set of passengers with their first flight delayed and M represent the set of passengers who missed their second, connecting flight.

(a) Complete the Venn diagram below to show this information. Complete all regions. (4 marks)



Solution
See diagram
Specific behaviours
✓✓✓✓ calculates correct value for each cell

(b) Determine: $n(D \cap \overline{M})$. (1 mark)

Solution
87
Specific behaviours
✓ determines the correct value

(c) If one of these passengers is selected at random, determine

(i) $P(\overline{D} \mid M)$. (2 marks)

Solution
$\frac{31}{96} = 0.3229$
Specific behaviours
✓ determines the correct numerator ✓ determines the correct denominator

(ii) $P(\overline{D \cap M})$. (2 marks)

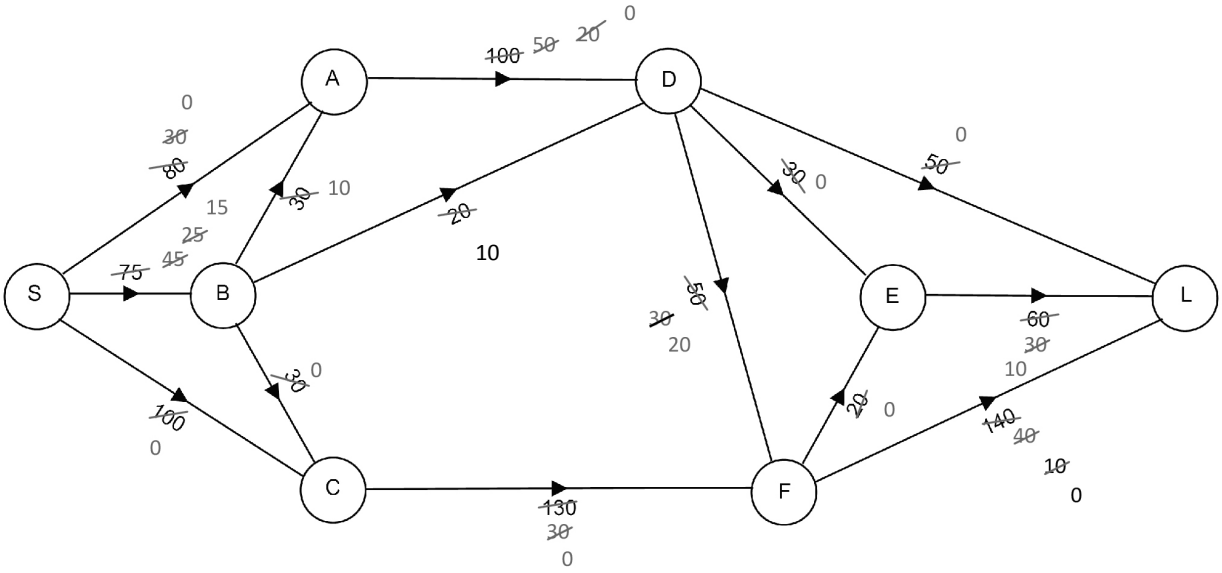
Solution
$\frac{505}{570} = 0.886$
Specific behaviours
✓ determines the correct numerator ✓ determines the correct denominator

(d) Determine the proportion of passengers who missed their second, connecting flight, even though their first flight was not delayed. (2 marks)

Solution
$\frac{31}{31+387} = \frac{31}{418} = 0.074$ (Accept 7.4%)
Specific behaviours
✓ determines the correct numerator ✓ determines the correct denominator

Question 15 **(7 marks)**

In a mining operation ore is moved from a central stockpile S to a loading station L through an ore processing plant consisting of six processors A, B, C, D, E and F linked by a system of conveyor belts. The network below displays the operation with the arcs representing the conveyor belts. The number on each arc represents the maximum amount of ore, in tonnes per minute, that can be moved along that conveyor belt.



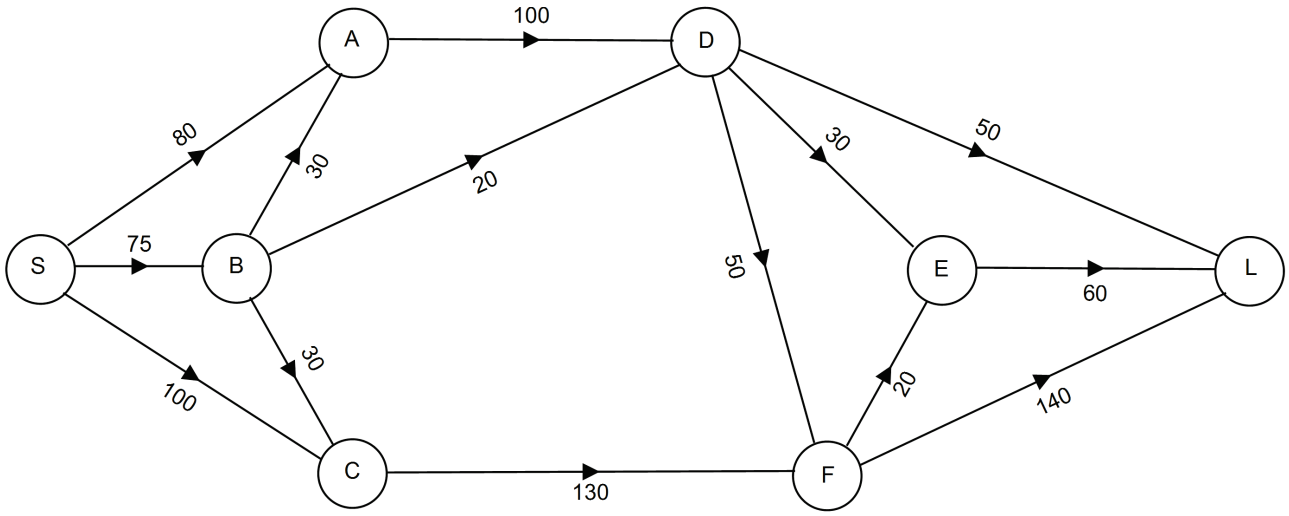
(a) What is the maximum amount of ore, in tonnes per minute, that can be moved from the stockpile S to the loading station L? Show systematic working to allow your solution to be checked.

(5 marks)

Solution	
S, C, F, L	: 100
S, B, C, F, F	: 30
S, A, D, L	: 50
S, A, D, E, F	: 30
S, B, A, D, F, E, L	: 20
S, B, D, F, L	: 10
TOTAL	: 240 tonnes per minute
Specific behaviours	
✓ determines at least three (3) paths with correct flow contribution	
✓ determines at least four (4) paths with correct flow contribution	
✓ determines at least five (5) paths with correct flow contribution	
✓ determines all paths with correct flow contribution	
✓ determines the correct maximum value	

(b) What effect, if any, would there be on the maximum flow of ore from S to L if the capacity of the conveyor belt FE was increased by 15 tonnes per minute? Justify your answer.

(2 marks)



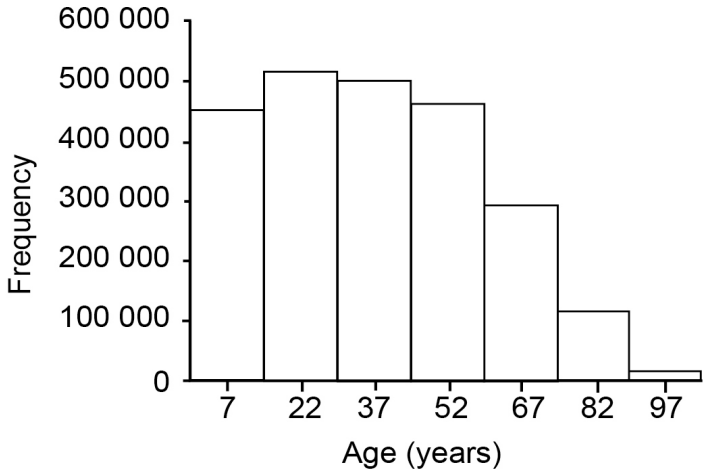
Solution
Additional flow of 10 tonnes per hour through path: S, B, D, F, E, L path enables extra flow (or alternative path according to answer in Part (a))
Specific behaviours
✓ determines the correct increase (according to answer to Part (a)) ✓ identifies path which allows for this increase (according to answer in Part (a)).

Question 16 **(11 marks)**

The estimated resident population of Western Australia at 30 June 2011, according to the Australian Bureau of Statistics (ABS), has been grouped into equal-sized intervals and these data are shown in the table and frequency histogram below.

Estimated resident population for Western Australia, at 30 June 2011

Age (years)	Frequency (number of persons)
0–14	453 747
15–29	515 339
30–44	502 452
45–59	462 003
60–74	286 187
75–89	114 436
90–104	12 246
Total	**2 346 410**



(a) For these data, determine

(i) the modal class. (1 mark)

Solution
15–29 years
Specific behaviours
✓ states the correct class

(ii) the largest possible range. (1 mark)

Solution
104
Specific behaviours
✓ states the correct range

(iii) the mean. (1 mark)

Solution
37.02 years (Accept 37)
Specific behaviours
✓ states the correct mean to an appropriate level of accuracy

(iv) the standard deviation. (1 mark)

Solution
22.25 years (Accept 22)
Specific behaviours
✓ states the correct standard deviation to an appropriate level of accuracy

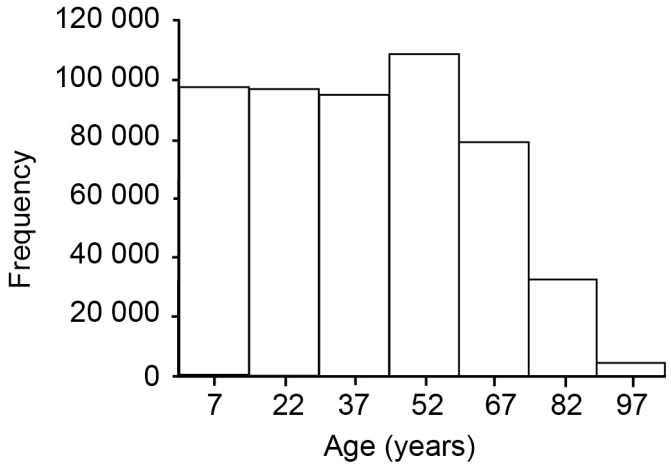
(v) the median class. Justify your answer. (2 marks)

Solution
30–44 years
Specific behaviours
✓ states the correct class ✓ identifies the correct reasoning (453 747 + 515 339 + 502 452 > ½ × (2 346 410))

The estimated resident population of Tasmania at 30 June 2011, according to the ABS, has been grouped into equal-sized intervals and these data are shown in the table and frequency histogram below. The mean age for this population is 39.48 years and the median class is 30–44 years.

Estimated resident population for Tasmania, at 30 June 2011

Age (years)	Frequency (number of persons)
0–14	97 694
15–29	96 220
30–44	94 446
45–59	107 789
60–74	78 337
75–89	32 710
90–104	3364
Total	**510 560**



(b) Does Western Australia or Tasmania tend to have ages that are more spread out? Justify your choice. (2 marks)

Solution
Tasmania tends to have ages that are more spread out. - greater standard deviation of 23.4 years - a higher proportion of scores are further above the mean, giving a greater spread.
Specific behaviours
✓ draws the correct conclusion ✓ identifies a valid reason for the greater spread

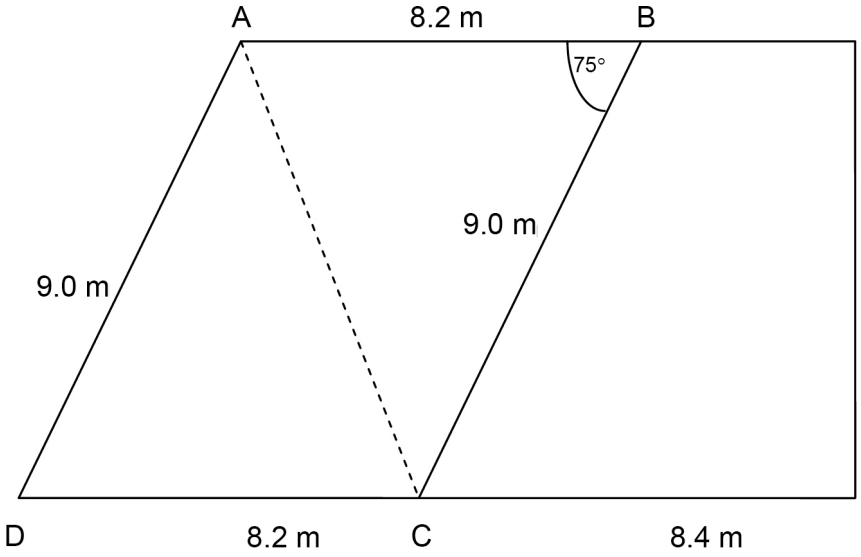
(c) A typical resident, in terms of their age, is to be chosen to represent each state. Should these typical residents be chosen from the same age intervals? Explain your answer.

(3 marks)

Solution
They should be from different age intervals. The modal classes are different for the two states: - Western Australia should select someone between 15–29 years - Tasmania should select someone between 45–59 years. (Even though the median classes are the same for both states, the mean age for Tasmania is also higher (39.48 years) than the mean for Western Australia, so an older person should be chosen.)
Specific behaviours
✓ chooses different age intervals ✓ identifies different modal classes ✓ identifies different means or details the use of modal classes

Question 17 **(10 marks)**

The Jones family is planning the back garden of their new house. The diagram below (not drawn to scale) shows the area. All dimensions are in metres and angles are in degrees.



The Jones family decides to build a limestone wall (one block high) from A to C to partition off the playground area ACD from the rest of the back garden.

(a) Using trigonometry, calculate the length of this wall. (3 marks)

Solution
$AC^2 = AB^2 + BC^2 - 2AB \times BC \cos 75°$ $= 67.24 + 81 - 38.2017 = 110.0383$ $\therefore \quad AC = 10.4899 \approx 10.5$ m
Specific behaviours
✓ applies cosine rule ✓ shows the correct substitution ✓ calculates the correct value of AC

(b) Limestone blocks come in 500 mm lengths. How many blocks will the Jones family need to buy? (2 marks)

Solution
Need 10.4899 m Require: 21 blocks
Specific behaviours
✓ shows the correct calculation ✓ gives correct rounding

(c) The playground area is to be covered by a shade sail with poles at A, C and D. Using trigonometry, determine

(i) the size of the angle ACD. (3 marks)

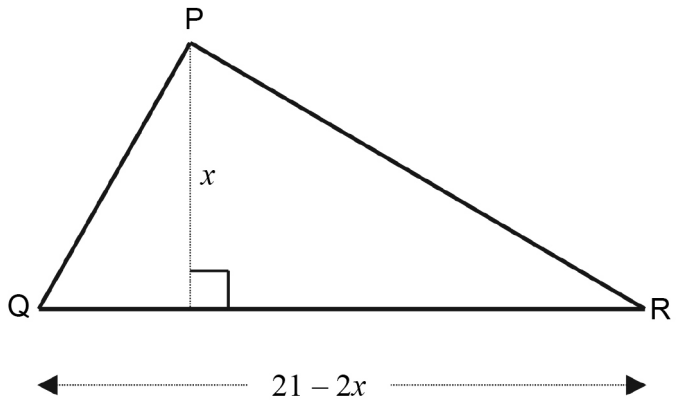
Solution
$\cos\angle ACD = \dfrac{(10.4899)^2 + (8.2)^2 - 9^2}{2(10.4899)(8.2)}$ $= \dfrac{96.2783}{172.0346} = 0.5596$ $\angle ACD = 55.9687° \simeq 55.97°$ Or correct application of the sine rule
Specific behaviours
✓ demonstrates the use of trigonometry ✓ calculates $\cos\angle ACD$ or $\sin\angle ACD$ ✓ states correct solution

(ii) the area of the playground ACD. (2 marks)

Solution
$\text{Area of } ACD = \dfrac{1}{2} \times 9 \times 8.2 \times \sin 75°$ $= 35.6427 \text{ m}^2$ $\simeq 35.64 \text{ m}^2$
Specific behaviours
✓ applies appropriate area formula ✓ determines the correct calculation of area

Question 18 **(10 marks)**

Triangle PQR has a base of $(21-2x)$ cm and perpendicular height of x cm, as shown in the triangle below.



(a) If $x = 4$, calculate the area of the triangle. (2 marks)

Solution
Area of PQR $= \frac{1}{2} \times 4(21-2\times 4) = 26 \text{ cm}^2$
Specific behaviours
✓ identifies base and height ✓ evaluates area correctly

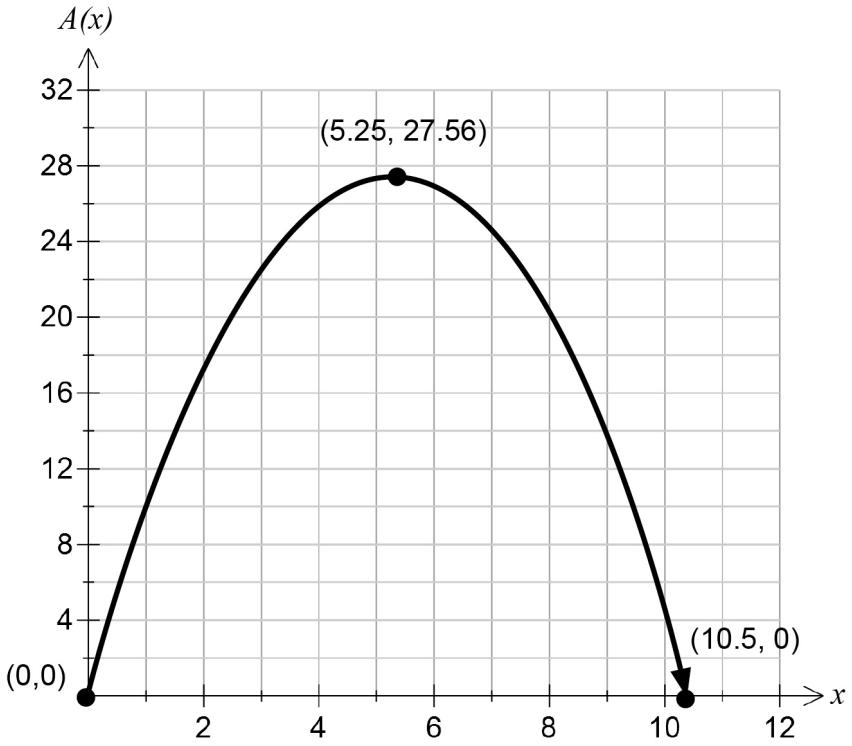
(b) Determine an expression for the area function of the triangle in terms of x, i.e. complete the following:

$A(x) =$ (1 mark)

Solution
$A(x) = \frac{1}{2}x(21-2x)$
Specific behaviours
✓ identifies the correct expression for $A(x)$

(c) Sketch the area function on the axes below. Label all significant features clearly.

(4 marks)

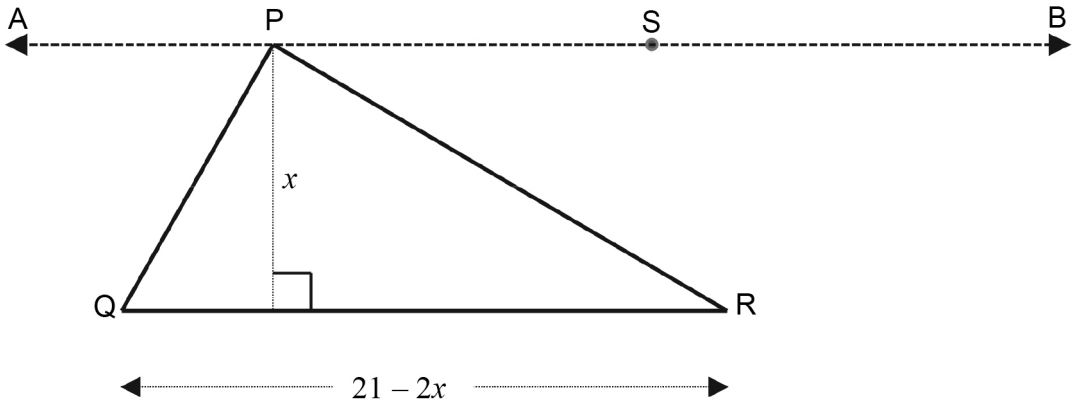


Solution
See graph
Specific behaviours
✓ draws the correct shape ✓ shows the $(0, 0)$ point ✓ shows the $(10.5, 0)$ point ✓ shows the maximum

(d) What is the maximum possible area of triangle PQR? (1 mark)

Solution
27.56 (consistent with answer to Part (c))
Specific behaviours
✓ determines the correct solution

(e) Line AB has been drawn parallel with base QR of the triangle and passing through point P.



(i) For the point S, on line AB, what is the maximum area of triangle QRS? (1 mark)

Solution
27.56
Specific behaviours
✓ calculates the correct area

(ii) If point S could be moved anywhere along line AB, describe how the location would affect the area of triangle QRS. (1 mark)

Solution
The area would not change with the location of point S
Specific behaviours
✓ identifies that triangle SQR has the same area as triangle PQR, regardless of the location point S

ACKNOWLEDGEMENTS

Section Two

Question 8 Data source: Independent Real Estate Consulting. (n.d.). *Agents fees, real estate fees, real estate commission – TAS, effective 1 July 2011*. Retrieved March 13, 2012, from http://irec.com.au/index.php?c=4.

Question 10 Data source: Australian Bureau of Statistics. (2012, February 23). *8146.0–Household use of information technology, Australia, 2010–11* [Excel spreadsheet]. Retrieved March, 2012, from www.abs.gov.au. Licensed under a Creative Commons Attribution 2.5 Australia licence.

Question 16 Data source: Australian Bureau of Statistics. (2011, December 19). *31010DO002_201106 Australian demographic statistics, June 2011* [Excel spreadsheet]. Retrieved March 27, 2012, from abs.gov.au. Licensed under a Creative Commons Attribution 2.5 Australia licence.